Mimicking Λ with a spin-two ghost condensate

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Aim

- To generate an effective cosmological constant from a pure gravity theory where the Lagrangian is only a function of curvature invariants using non-conventional branches *a la* DGP/Khoury Fading gravity model

$$\mathcal{L} = \sqrt{-g}\, f(R, R^{ab}R_{ab}, R^{abcd}R_{abcd}......)$$

- Assume some unknown symmetry/principle sets fundamental Λ to zero.
- This is achieved in the DGP model, but at a heavy price: *massive gravitons, modification of gravity at large scales, ghost issues.*
- Inspiration: String theory admits AdS solutions without the introduction of matter, Gubser + Friess (`hep-th/0512355`) have shown these to be present at all orders in α'.
- By introducing an orbifold fixed plane, a negative Λ in 5d can be converted to a positive Λ in 4d.

Einstein-Hilbert + Gauss Bonnet

S. Deser and Z. Yang (1989)

For example, in 5d, unique terms that maintain conventional Cauchy problem for gravity are Einstein-Hilbert plus the Gauss-Bonnet invariant, a specific combination of R^2 terms.

$$S = \frac{1}{2\kappa_5^2} \int \mathrm{d}^5 x \sqrt{-g} \left[+\mathrm{R} + \frac{\ell^2}{2} \mathcal{R}_2 \right]$$

- Admits AdS vacuum with curvature length scale ℓ.
- But AdM mass is negative, highly unstable classically and quantum mechanically.
- Basic reason, graviton kinetic energy has wrong sign w.r.t. matter: gravitons are ghosts.

Ghost-condensates

N. Arkani-Hamed et *al.* hep-th/0312099

- The quadratic Lagrangian for ϕ has the wrong-sign kinetic term

$$\mathcal{L}_\phi = +\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{4M^4}\left(\partial_\mu\phi\,\partial^\mu\phi\right)^2$$

which means that the usual background with $\langle\phi\rangle = 0$ is unstable.

- Higher order terms can stabilize a background where $\langle\dot{\phi}\rangle = M^2$. Lorentz invariance is thus broken.
- Instead we consider the same mechanism in 5d

$$\mathcal{L}_\phi = +\frac{1}{2}\partial_A\phi\partial^A\phi - \frac{\theta}{4}(\partial_A\phi\partial^A\phi)^2,$$

where the background solution has a constant "velocity" along the extra-direction $\langle\partial_y\phi\rangle = \sigma/\sqrt{\theta}$.

- The action for the perturbations is

$$\delta\mathcal{L}^{(2)}_\phi = -\frac{1}{2}\left((\sigma^2-1)\partial_\mu\delta\phi\,\partial^\mu\delta\phi + (3\sigma^2-1)(\partial_y\delta\phi)^2\right).$$

The model

- We take a gravitational version of the ghost condensate

$$S = \frac{1}{2\kappa_5^2}\int \mathrm{d}^5x\sqrt{-g}\left[-\mathrm{R}-\frac{\ell^2}{2}\mathcal{R}_2\right] + \int \mathrm{d}^4x\sqrt{-q}\left[\frac{\alpha}{2\kappa_4^2}R+\mathcal{L}_m-\frac{1}{\kappa_5^2}Q\right]$$

- Expanding around the canonical vacuum (Minkowski space) leads to instabilities when matter is present in the bulk (gravitons are ghosts).
- The unconventional sign for the EH term is balanced out by GB term so that perturbations around the AdS background solution have a positive kinetic energy.
- Q is the generalization of the Gibbons-Hawking boundary term necessary to make the variational problem well behaved (cubic in extrinsic curvature K).

Accelerating solution

- Take bulk solution to be AdS in the de Sitter slicing

$$ds^2 = \frac{\ell^2 H^2}{\sinh^2 Hy}(dy^2 - dt^2 + e^{2Ht} d\vec{x}^2)$$

- Assuming brane at $y = \bar{y}$, Israel junction condition gives

$$\frac{1}{\ell}\sqrt{1+\ell^2 H^2}\left(1 + 4\ell^2 H^2\right) = -\frac{\kappa_5^2}{6}\rho + \frac{\alpha}{2}\frac{\kappa_5^2}{\kappa_4^2}H^2$$

- No solution with $\rho = 0$ and $H = 0$.
- Provided $\gamma = \frac{3\alpha}{2}\frac{\kappa_5^2}{\ell\kappa_4^2} > \gamma_c \approx 7$, the brane geometry accelerates in the absence of any stress-energy or cosmological constant on the brane or bulk.
- Modified Friedmann equation:

$$H^2 = \frac{\kappa_4^2}{3}\left(\Lambda_{\mathrm{eff}} + \rho + \mathcal{O}(\rho^2/\rho_0)\right).$$

where $\Lambda_{\mathrm{eff}} \sim \frac{2}{\alpha\kappa_5^2\ell}$ and $\rho_0 \sim 1/\ell^2\kappa_4^2$.

Tensor perturbations

- Considering tensor perturbations in the vacuum,

$$\left[\partial_y^2 + m^2 - \left(\frac{9}{4} + \frac{15}{4\sinh^2 Hy}\right) H^2\right] u_m(y) = 0$$

$$\left[\partial_y + \frac{3H}{2}\coth H\bar{y} + \left(\frac{\alpha\kappa_5^2}{2\kappa_4^2} - 2\ell^2 H \coth H\bar{y}\right) m^2\right] u_m(\bar{y}) = 0$$

m^2 being the eigenvalue of the operator $\left[\Box - 2H^2\right]$.

- The graviton zero mode is massless $m = 0$ with profile

$$u_0(y) = A(\sinh Hy)^{-3/2} \sim a(y)^{3/2}.$$

- The remaining KK modes are massive with $m^2 > \frac{9}{4}H^2$.
- Perturbations behave similarly as in the stable branch of DGP.
- Theory is ghost free and stable

Application to cosmology

- The main motivation is to use the emergent acceleration to describe the current cosmic acceleration

$$\Lambda_{\text{eff}} \sim \frac{1}{\kappa_5^2 \ell} \sim \frac{1}{\kappa_4^2 l_H^2}.$$

- Requiring conventional Friedmann eq. at nucleosynthesis time ($\rho_n \sim \left(10^{-3}\text{GeV}\right)^4$) leads to the additional constraint

$$\ell \ll \ell_c = H_n^{-1} \sim 1/\kappa_4 \sqrt{\rho_n} \sim 10^{25}\,\text{GeV}^{-1} \sim 10^3 \text{km}.$$

- Using this constraint, Λ_{eff} has the right order of magnitude if

$$\gamma \sim \frac{\alpha \kappa_5^2}{\ell \kappa_4^2} \gg \frac{l_H^2}{\ell_c^2} \sim 10^{36}.$$

- Apparent concern: 5d Planck scale is low $\sim 10eV$, sounds disastrous? However, effective brane stress energy is $T_{\mu\nu}^{eff} = T_{\mu\nu} - \frac{\alpha}{\kappa_4^2} G_{\mu\nu} \sim \frac{1}{\gamma} T_{\mu\nu} \sim 10^{-36} T_{\mu\nu}$.

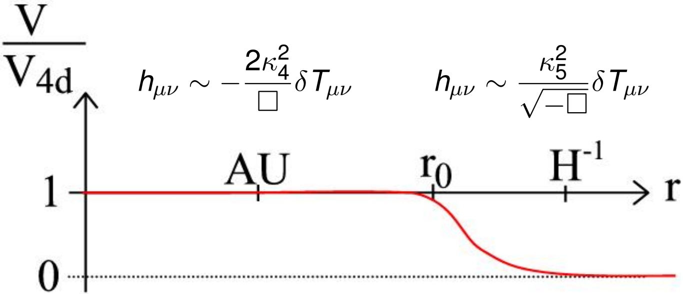
Modification of Gravity at subhorizon scales

- At subhorizon scales, we can neglect H in the study of the gravitational response to a matter source $\delta T_{\mu\nu}$ on the brane.
- For $r \ll H^{-1}$, the induced metric perturbation on the brane in de Donder gauge is

$$\bar{h}_{\mu\nu} = -\frac{2\kappa_4^2}{\Box}\left(\delta T_{\mu\nu} - \frac{1}{2}\delta T \gamma_{\mu\nu}\right) + \frac{K_0(\sqrt{-\Box}\ell)}{\frac{\kappa_5^2}{\ell\kappa_4^2}K_1(\sqrt{-\Box}\ell) + \sqrt{-\Box}\ell\left(\frac{\gamma}{3} - 2\right)K_0(\sqrt{-\Box}\ell)}\frac{\ell\kappa_4^2}{\sqrt{-\Box}}\Sigma_{\mu\nu}.$$

where $\Sigma_{\mu\nu} = \delta T_{\mu\nu} - \frac{1}{3}\delta T \gamma_{\mu\nu} + \frac{1}{3\Box}\delta T_{;\mu\nu}$.

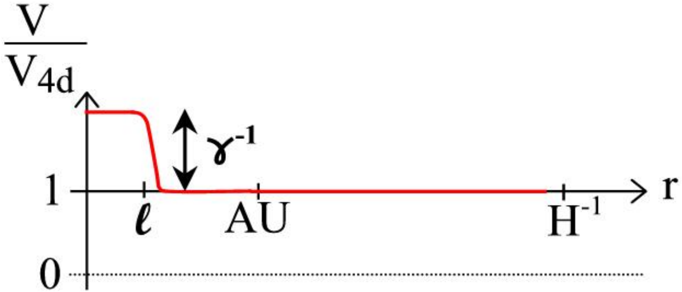
- In the DGP model, gravity behaves as



$$h_{\mu\nu} \sim -\frac{2\kappa_4^2}{\Box - \frac{\sqrt{-\Box}}{r_0}}\delta T_{\mu\nu}$$

where $r_0 = \kappa_5^2/\kappa_4^2$.

- Whereas in the present model...



$r \ll \ell$

$$h_{\mu\nu} \sim -\frac{2\left(1+\frac{3}{2\gamma}\right)\kappa_4^2}{\Box}\delta T_{\mu\nu}$$

$r \gg \ell$

$$h_{\mu\nu} \sim -\frac{2\kappa_4^2}{\Box}\delta T_{\mu\nu}$$

Summary

- A naively ghostly theory with a negative EH action and additional GB terms can admit a stable AdS vacuum.
- Compactifying on an orbifold with only curvature couplings on the brane gives rise to de Sitter solutions in absence of brane tensions/bulk c.c.
- This solution is stable, no ghosts or tachyons, and the normalizable graviton zero mode is massless.
- Demanding a low-energy behaviour at the time of nucleosynthesis and the right order of magnitude for Λ_{eff} requires a hierarchy between 4d and 5d gravitational couplings.
- Gravity is effectively 4 dimensional throughout most of energy regime, bulk action serves largely to generate effective cosmological constant.

Open questions / Future directions

- Beyond linear perturbations ?

  Indications are that low energy physics is still conventional as expected from massless graviton.

- Quantum corrections and how does Λ_{eff} flow under the Renormalization Group ? N.B. 5d energy scales are very low $< 10eV$.

  Since there are a continuum of KK massive gravitons, and mass gap is Hubble scale, RG flow probably has to be dealt with in 5d theory

- Can one tunnel from the stable to ghostly branch?
- Are there generalizations of this type of mechanism?