Inflation and Our Miniverse

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WHY A MINIVERSE*?

(* aka, a "small Universe")

See: A. Linde, JCAP 10 (2004) 004

1) COMPACT SPATIAL SECTIONS

$\Rightarrow$ A DISCRETE SPECTRUM OF

FLUCTUATIONS $\Rightarrow$ LOW-ℓ

SUPPRESSION?

Good reviews:

Lachieze-Rey & Luminet, Phys. Rep. 254 (1995) 136

J. Levin, Phys. Rep. 365 (2002) 251

2) IF $L \lesssim H^{-1}$, THEN THE WHOLE UNIVERSE

IS CAUSALLY CONNECTED

(MAYBE CHAOTIC MIXING?)

Cornish et al., Phys. Rev. Lett. 77 (1996) 215

* 3) QUANTUM COSMOLOGY

FINITE VOLUME OF SPATIAL SECTIONS

Zeldovich & Starobinsky, Sov. Astron. Lett. 10 (1984) 135

* 4) IF $L < H^{-1}$, THE WHOLE UNIVERSE IS HOMOGENEOUS ON THE SCALE H^{-1}

$\Rightarrow$ HELPS WITH INITIAL CONDITIONS FOR INFLATION?

See: A. Linde, JCAP 10 (2004) 004

Quantum Cosmology

MiniSuperspace:

$$ds^2 = -N^2(t)\,dt^2 + a^2(t)\,d\Sigma_3^2$$

where $\Sigma_3 \subset \mathbb{E}^3,\ \mathbb{H}^3,$ or S^3

and $Vol(\Sigma_3) \equiv \mathcal{V}_\Sigma$ (Coord. 3-volume)

Action: $S = \mathcal{V}_\Sigma\, \bar{s}$, $\bar{s} \equiv$ Action per unit vol.

$$\Rightarrow \quad e^{i\frac{S}{\hbar}} = e^{i\frac{\mathcal{V}_\Sigma}{\hbar}\bar{s}}$$

Or, from the Wheeler–DeWitt Eq'n (with c.c.)

$$\left[-\frac{\partial^2}{\partial a^2} + U(a)\right]\Psi(a) = 0$$

$$U(a) = (6\mathcal{V}_\Sigma)^2\left(ka^2 - \frac{\Lambda}{3}a^4\right)$$

NOTE: V_Σ CAN BE HIDDEN BY PARAMETER REDEFINITIONS, BUT IT IS <u>PHYSICAL</u>.

Ex. SCH. EQ'N

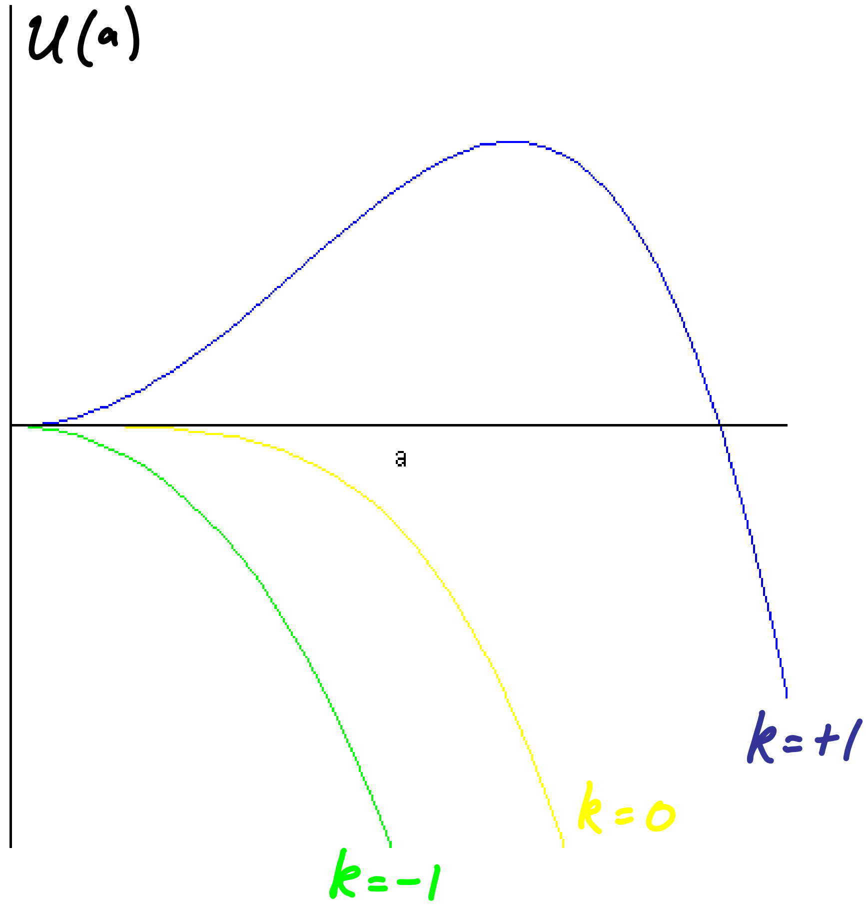
$$-\frac{\hbar^2}{2m}\nabla^2\psi_n + (V - E_n)\psi_n = 0$$

$\hbar \to 0$ or $m \to \infty$ is the "classical" or WKB limit, even though I can choose units such that $\hbar = m = 1$.

CONCLUSION:

FOR $V_\Sigma \to \infty$, QM BECOMES IRRELEVANT

(Note: Not everyone agrees with this!)



So, as $V_q \to \infty$

$\Rightarrow$ FOR $k=+1$ THERE IS NO TUNNELING AND FOR $k=-1,0$, THERE IS ONLY CLASSICAL EVOLUTION FROM THE SINGULARITY.

BUT QUANTUM COSMOLOGY WAS SUPPOSED TO SAVE US FROM THE INITIAL SINGULARITY!

Can Finite V_Ω and Quantum Cosmology Help in Low Scale Inflation?

A. Linde, loc. cit.

Standard Lore:

To get inflation, we need a smooth patch of size $R_H \sim \frac{1}{H}$

For low scale inflation, $H \ll 1$

$\Rightarrow$ $R_H \gg 1$ and we need to nucleate a large universe!

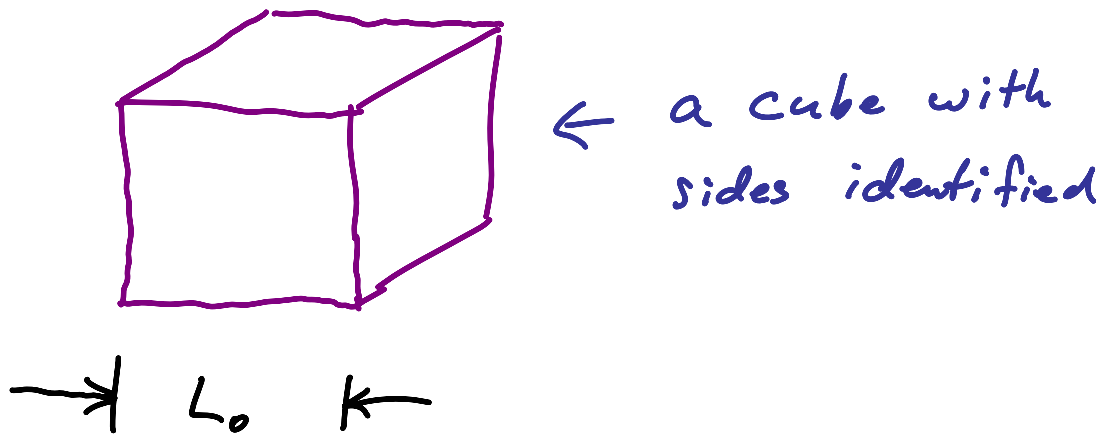
Not True!

$L = a L_0$ and $R_H \sim \frac{1}{H}$ are a priori independant.

$L = a L_0$: Global feature

$R_H \sim \frac{1}{H}$: Local feature

EXAMPLE: FLAT HYPERTORUS



WE CAN TILE $\mathbb{E}^3$ WITH CUBES OF ANY SIZE, SO WE CAN HAVE ANY SIZE UNIVERSE REGARDLESS OF THE HUBBLE PARAMETER.

NOTE: I CARE BECAUSE I WANT THE CLASSICAL EVOLUTION TO BEGIN ONLY WHEN $H \ll 1$

But...

There are Problems!

1) The HyperTorus is a bad example.

$\Rightarrow$ S^3 and $\mathbb{H}^3$ have **rigid** tilings

L and R_H are not independant

for most 3-surfaces of const. curve.

(see the reviews loc. cit.)

Example: S^3/I^* = Poincare Dodecahedron

120 tile S^3 each with volume: $\frac{V(S^3)}{120}$

Since $V(S^3)$ is determined by H^{-1}, so

is $V(S^3/I^*)$ and $L \sim \left(V(S^3/I^*)\right)^{1/3}$.

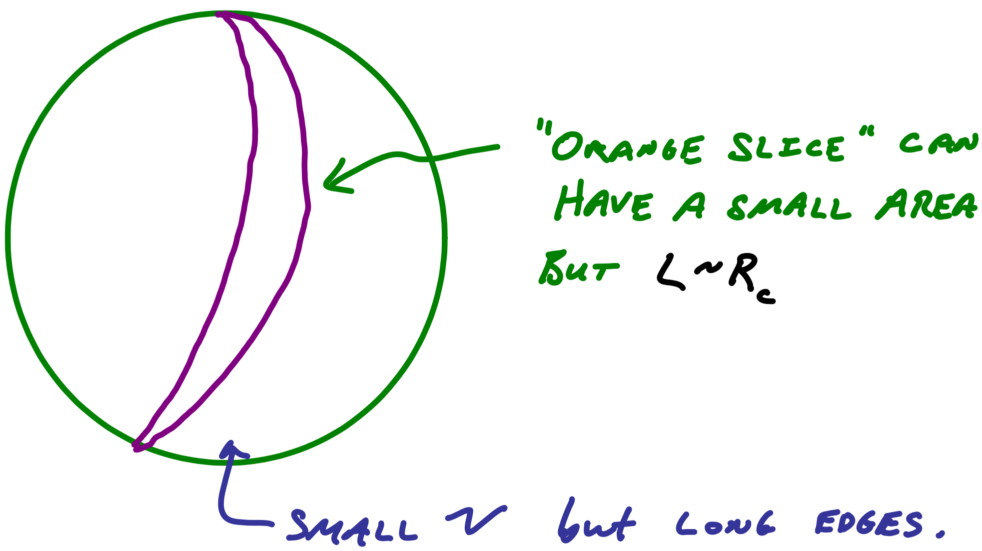
2) MANY SPACES HAVE SMALL VOLUMES, BUT THEIR LINEAR SIZE IS LARGE.

THESE SPACES ARE LIKE LONG CIGARS.

$\Rightarrow$ WE STILL NEED CORRELATIONS OVER LENGTHS $\sim R_H$!

E.G. "LENSE" SPACES : $S^3/Z_n \Rightarrow V_n \propto \frac{1}{n}$

BUT $L \sim R_H$ ALWAYS



(THANKS TO J. DISTLER FOR PROOF IN TERMS OF MODES.)

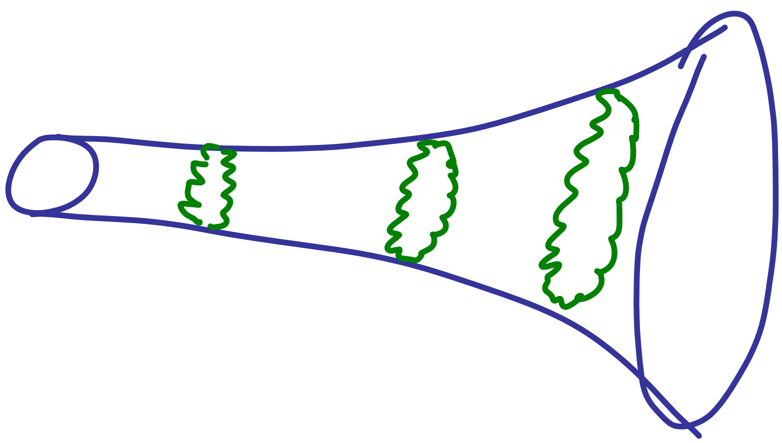
3) CASIMIR EFFECT:

NAIVELY: $T^{\mu\nu} \propto \frac{1}{a^4} \bar{g}^{\mu\nu}$

IN FACT, MOST SPACES ARE GLOBALLY ANISOTROPIC AND INHOMOGENEOUS

$\Rightarrow$ THESE SPACES ARE NOT STABLE.

Example: "Cuspy" $\mathbb{H}^3$



GLOBAL ANISOTROPY DRIVES THE SYSTEM AWAY FROM LOCAL FRW!

Conclusion:

Low scale inflation and quantum creation of the universe favor flat compactifications with length scales in all three directions that are similar.

(Perhaps this favours light, conformally coupled fields? or SUSY?)

↑ Mentioned by Linde